An optimal linear transformation for data assimilation

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Abstract

Estimation of extremely large covariance matrices and their use computationally to update prior state estimates (whether through minimization or solution of linear systems) are key difficulties in data assimilation for geophysical systems. It is often helpful to apply linear transformations of the state or observations, for example to facilitate the implementation of multivariate spatial covariances or improve the conditioning of the numerical problem. In particular, recent literature on multiscale ensemble Kalman filters can be cast as a search for transformations to coordinates in which localization of sample covariances is especially straightforward and effective. This leads to two questions: What linear transformation yields the simplest form of the update equations for the Kalman filter or best linear unbiased estimator? And what linear transformation is optimal to precede localization in ensemble Kalman filters? Answers to these questions appear to be the same. The resulting transformation diagonalizes the update, in the sense that all covariances become the identity and each coordinate in the observation space depends on a single, unique coordinate in the state space. The number and signal-to-noise ratio of the observations in the transformed space then characterize the update, including measures of information content for the observations, the condition number of the minimization problem, and the effects of sampling error from ensemble techniques.

Keywords: Kalman filter, covariance localization, information theory

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