
Randomised preconditioning in variational data assimilation

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Abstract

In variational data assimilation, the analysis can be obtained by minimising a series of linear least-squares cost functions. Each minimisation (called an inner loop) can be performed by using the conjugate gradient (CG) method, which needs preconditioning for fast convergence. In previous work, limited memory preconditioners (LMPs) have been constructed using approximations of the eigenvalues and eigenvectors of the Hessian of the cost function from the previous inner loop. If the Hessian changes significantly between consecutive inner loops, the LMP may perform poorly. To circumvent this, we propose using randomised methods for low rank eigenvalue decomposition to construct LMPs using information from the current inner loop. These methods can be easily parallelised and the cost can be comparable to just a few CG iterations if there are enough computational resources available. We compare three randomised methods when minimising the quadratic cost function arising in incremental weak constraint 4D-Var. Numerical experiments in idealized systems show that the resulting randomised LMPs perform better than the existing deterministic LMPs: a randomised LMP gives faster minimisation than a deterministic LMP, even when the former is constructed with fewer eigenvectors than the latter. Using these methods may allow more efficient and robust implementations of variational data assimilation methods.

Keywords: variational data assimilation, weak constraint 4DVar, preconditioning, randomised methods

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